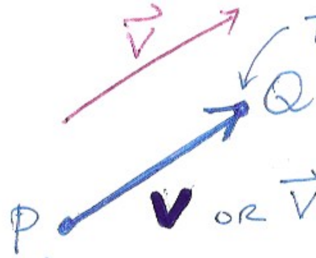
12.2

VECTORS

A DIRECTED LINE SEGMENT (DETERMINED BY DIRECTIO
MAGNITUDE)

TERMINAL POINT



$\mathbf{v}$ OR $\vec{v}$ OR $\overrightarrow{PQ}$

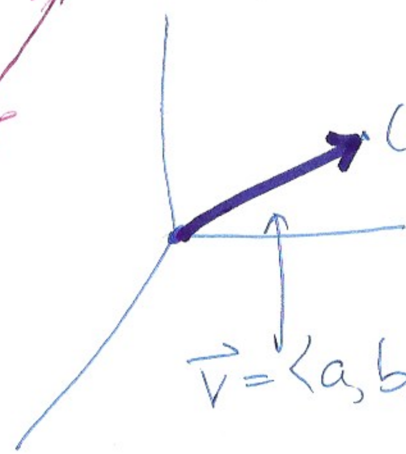
(NOT $\overleftarrow{QP}$)

INITIAL POINT

$\overrightarrow{PQ} \neq \overrightarrow{QP}$

IN 3D: $\vec{v} = \langle a, b, c \rangle$ IF ITS INITIAL POINT IS AT THE ORIGI
+ ITS TERMINAL POINT IS (x, y, z)

COMPONENT FORM



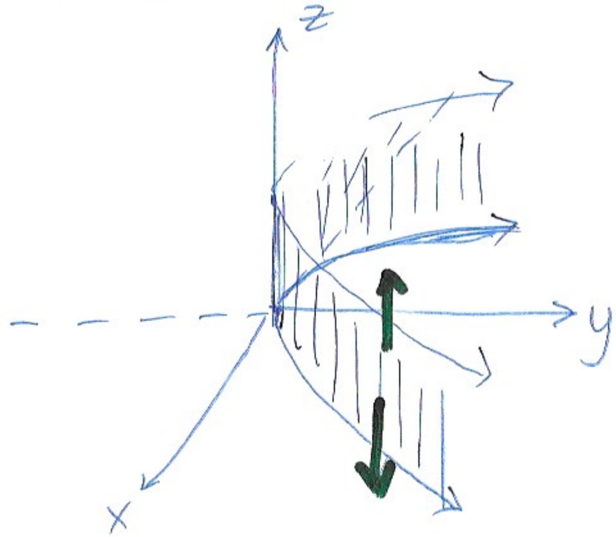
$\vec{v} = \langle a, b, c \rangle$

$\vec{v} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$
IF THE INITIAL POINT
IS (x_0, y_0, z_0)
AND THE TERMINAL POINT
IS (x_1, y_1, z_1)

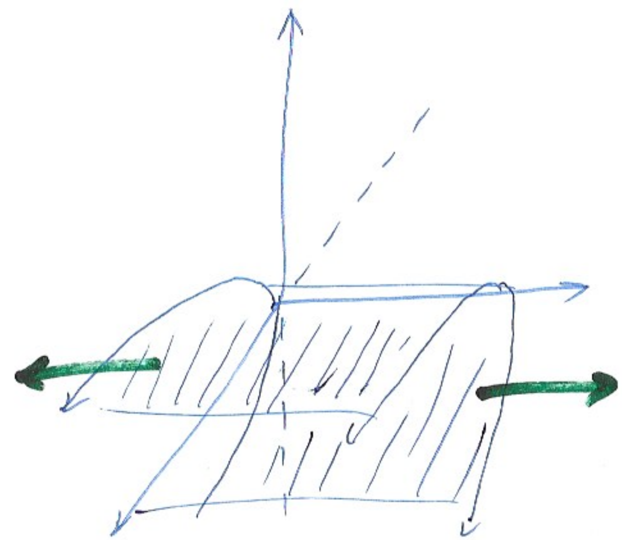
GRAPHS OF CYLINDERS

↳ IN 3D, EQ'N WITH 1 VARIABLE MISSING

$y = x^2$



$x = z^2$



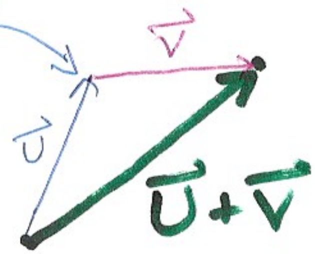
VECTOR OPERATIONS

$\vec{u}, \vec{v}$ $\qquad k \in \mathbb{R}$

$\vec{u}+\vec{v}$

$\boxed{\text{VECTOR ADDITION}}$

INITIAL POINT OF $\vec{v}$ = TERMINAL POINT OF $\vec{u}$

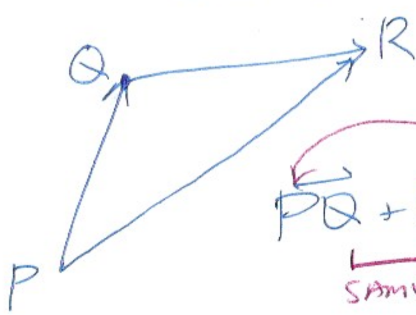


INITIAL POINT OF $\vec{u}$ TO TERMINAL POINT OF $\vec{v}$

IF $\vec{u}=\langle u_1, u_2, u_3\rangle$

AND $\vec{v}=\langle v_1, v_2, v_3\rangle$

THEN $\vec{u}+\vec{v}=\langle u_1+v_1, u_2+v_2, u_3+v_3\rangle$



$\overrightarrow{PQ}+\overrightarrow{QR}=\overrightarrow{PR}$

SAME

SCALAR MULTIPLICATION

$k\vec{v}$

$\vec{v}$

$2\vec{v}$

$\frac{1}{2}\vec{v}$

2 × MAGNITUDE
OF $\vec{v}$,
SAME DIRECTION
$(k>0)$

$\frac{1}{2}$ × MAGNITUDE
" "
" "

$(-1)\vec{v}$

$(-2)\vec{v}$

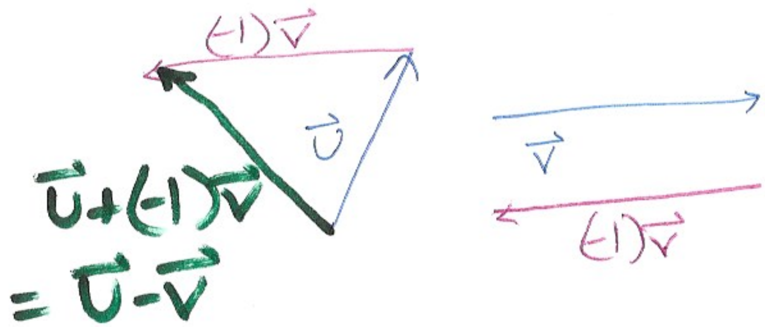
1 × MAGNITUDE
OF $\vec{v}$,
OPPOSITE DIRECTION
$(k<0)$

IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$

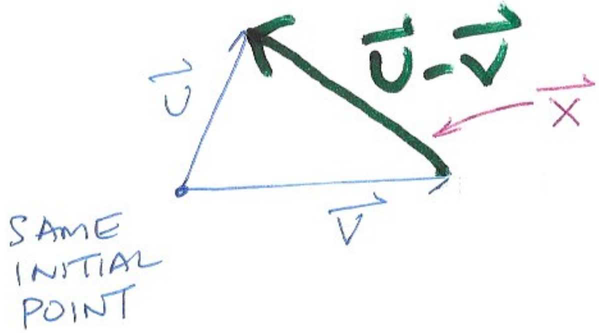
THEN $k\vec{v} = \langle kv_1, kv_2, kv_3 \rangle$

VECTOR SUBTRACTION

$$\vec{u} - \vec{v} = \vec{u} + (-1)\vec{v}$$



OR



$$\vec{v} + \vec{x} = \vec{u}$$
$$\vec{x} = \vec{u} - \vec{v}$$

MAGNITUDE OF $\vec{v} = \|\vec{v}\|$

IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

IF $P=(1,-1,-2)$, $Q=(0,2,1)$, $R=(-1,0,2)$

AND $\vec{U}=\overrightarrow{PQ}$, $\vec{V}=\overrightarrow{PR}$

① FIND $\|\vec{V}\|$

$$\vec{V}=\langle -1-1,\ 0-{}^-1,\ 2-{}^-2\rangle = \langle -2,1,4\rangle$$

$$\|\vec{V}\| = \sqrt{(-2)^2+1^2+4^2} = \sqrt{4+1+16} = \sqrt{21}$$

② FIND $2\vec{U}-3\vec{V}$

$$\vec{U} = \langle 0-1,\ 2-{}^-1,\ 1-{}^-2\rangle = \langle -1,3,3\rangle$$

$$2\vec{U}-3\vec{V} = 2\langle -1,3,3\rangle - 3\langle -2,1,4\rangle$$

$$= \langle -2,6,6\rangle - \langle -6,3,12\rangle$$

$$= \langle -2-{}^-6,\ 6-3,\ 6-12\rangle$$

$$= \langle 4,3,-6\rangle$$

$\vec{0} = \langle 0,0,0 \rangle$

$\|\vec{0}\| = 0$

IF $\|\vec{u}\| = 0$, THEN $\vec{u} = \vec{0}$

UNIT VECTORS

$\vec{v}$ IS A UNIT VECTOR IF AND ONLY IF $\|\vec{v}\| = 1$

IS $\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle$ A UNIT VECTOR? $\boxed{\text{NO}}$

$$\left\| \left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle \right\| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3} \neq 1$$